1 Math stuff

Cubic interpolation for one segment $[x_k, x_{k+1}]$ can be described as:

$$
\begin{aligned}
f(t) &= c_{oef1}t^3 + c_{oef2}t^2 + c_{oef3}t + c_{oef4} \qquad with \\
t(x) &= \frac{x - x_k}{x_{k+1} - x_k}
\end{aligned}
$$

and

$$
\begin{aligned}
c_{oef1} &= 2p_0 - 2p_1 - m_0 - m_1 \\
c_{oef2} &= -3p_0 + 3p_1 - 2m_0 - m_1 \\
c_{oef3} &= m_0 \\
c_{oef4} &= p_0
\end{aligned}
$$

(see Wikipedia-Links below)

If we rewrite this as function of $d = x - x_k$ we get

$$
\begin{aligned}
f'(d) &= c'_{oef1}d^3 + c'_{oef2}d^2 + c'_{oef3}d + c'_{oef4} \qquad with \\
c'_{oef1} &= \frac{c_{oef1}}{(x_{k+1} - x_k)^3} \\
c'_{oef2} &= \frac{c_{oef2}}{(x_{k+1} - x_k)^2} \\
c'_{oef3} &= \frac{c_{oef3}}{x_{k+1} - x_k} \\
c'_{oef4} &= c_{oef4}
\end{aligned}
$$

The implemented algorithm uses two helper variables to calculate the coefficients of f' efficiently:

$$
\begin{aligned}
common = m_k + m_{k+1} - 2\frac{p_{k+1} - p_k}{x_{k+1} - x_k} \\
invLength = \frac{1}{x_{k+1} - x_k}
\end{aligned}
$$

We use $p_0 = p_k$, $p_1 = p_{k+1}$, $m_0 = m_k(x_{k+1} - x_k)$, $m_1 = m_{k+1}(x_{k+1} - x_k)$ and $s = \frac{p_{k+1} - p_k}{x_{k+1} - x_k}$. The tangents are scaled with the length of the segment.

If we insert this into the equations for the coefficients we get the formulas that are used in the algorithm:

$$
\begin{aligned}
c'_{oef1} &= \frac{c_{oef1}}{(x_{k+1}-x_k)^3} \\
&= \frac{2p_0 - 2p_1 + m_0 + m_1}{(x_{k+1}-x_k)^3} \\
&= (2p_k - 2p_{k+1} + m_k(x_{k+1}-x_k) + m_{k+1}(x_{k+1}-x_k)/(x_{k+1}-x_k)^3 \\
&= \frac{(2p_k - 2p_{k+1} + m_k(x_{k+1}-x_k) + m_{k+1}(x_{k+1}-x_k)}{x_{k+1}-x_k}/(x_{k+1}-x_k)^2 \\
&= (\frac{2p_k - 2p_{k+1}}{x_{k+1}-x_k} + m_k + m_{k+1}) * invLength^2 \\
&= (-2\frac{p_{k+1}-p_k}{x_{k+1}-x_k} + m_k + m_{k+1}) * invLength^2 \\
&= common * invLenght^2
\end{aligned}
$$

$$
\begin{aligned}
c'_{oef2} &= \frac{c_{oef2}}{(x_{k+1}-x_k)^2} \\
&= (-3p_0 + 3p_1 - 2m_0 - m_1)/(x_{k+1}-x_k)^2 \\
&= (-3p_k + 3p_{k+1} - 2 * m_k(x_{k+1}-x_k) - m_{k+1}(x_{k+1}-x_k))/(x_{k+1}-x_k)^2 \\
&= (\frac{-3p_k + 3p_{k+1}}{x_{k+1}-x_k} - 2m_k - m_{k+1}) * invLenght \\
&= (3\frac{p_{k+1}-p_k}{x_{k+1}-x_k} - 2m_k - m_{k+1}) * invLenght \\
&= (\frac{p_{k+1}-p_k}{x_{k+1}-x_k} + 2\frac{p_{k+1}-p_k}{x_{k+1}-x_k} - m_k - m_{k+1} - m_k) * invLenght \\
&= (s - common - m_k) * invLenght
\end{aligned}
$$

$$
\begin{aligned}
c'_{oef3} &= \frac{c_{oef3}}{x_{k+1}-x_k} \\
&= \frac{m_0}{x_{k+1}-x_k} \\
&= \frac{m_k(x_{k+1}-x_k)}{x_{k+1}-x_k} \\
&= m_k
\end{aligned}
$$

$$
c'_{oef4} = c_{oef4} = p_0 = p_k
$$

2 Useful Links

http://de.wikipedia.org/w/index.php?title=Kubisch_Hermitescher_Spline&
oldid=130168003)
http://en.wikipedia.org/w/index.php?title=Monotone_cubic_interpolation&
oldid=622341725
http://math.stackexchange.com/questions/45218/implementation-of-monotone-cubic-interpolati
http://math.stackexchange.com/questions/4082/equation-of-a-curve-given-3-points-and-addit
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